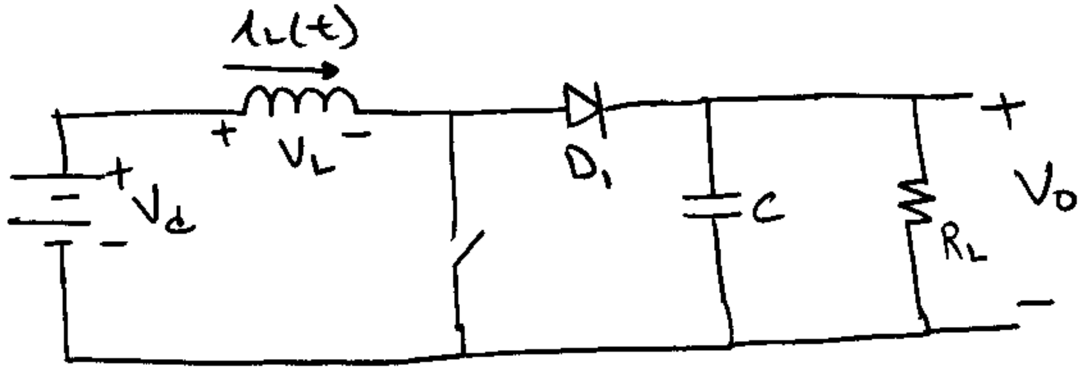
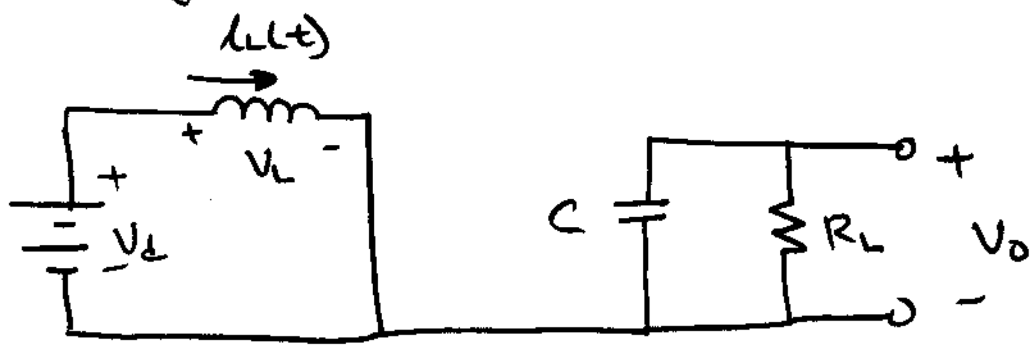
Boost Converter

(Step-Up) converter

$V_o > V_d$

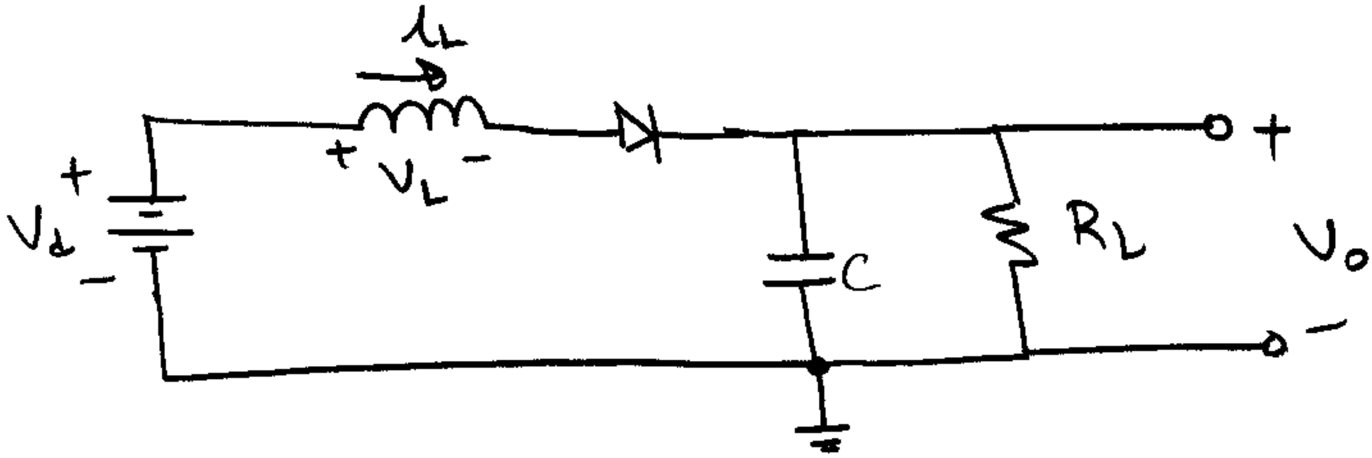


During ton: switch closed, D_1 = off



- Inductor charges from V_d
- Capacitor supplies power to Load
- $V_L = V_d$
- Inductor current increases

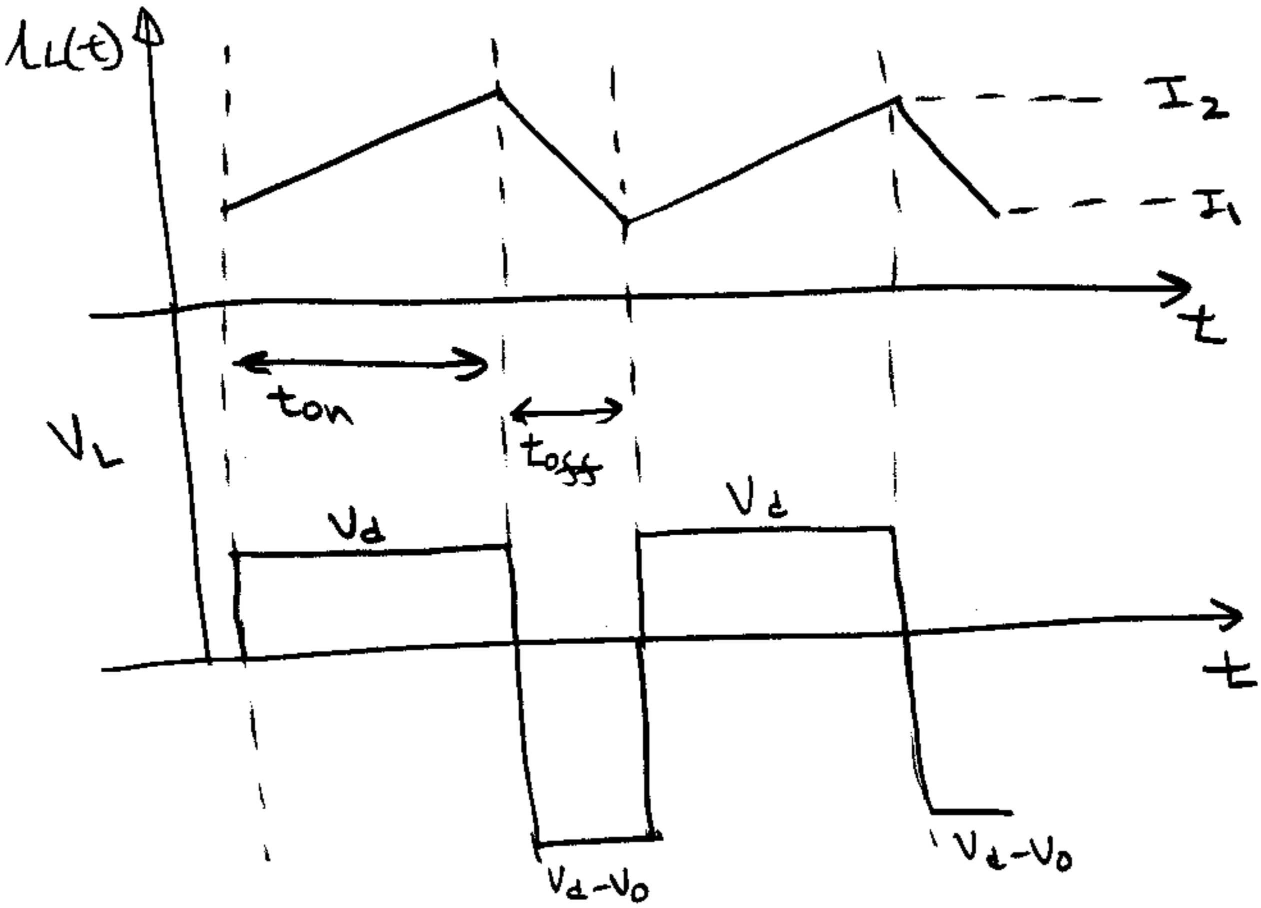
During t_{off}: switch open, D_1 = on



- Assume diode is ideal
- $V_L = V_d - V_o$; since $V_o > V_d$, the inductor current will decrease

- Inductor sources power to C and R_L

Continuous mode



$T_s = t_{on} + t_{off}$

Since in steady state, the integral of $V_L = 0$ over one cycle

$$\int_{T_s} V_L(t)\,dt = 0$$

$$\Rightarrow (V_d\, t_{on}) + (V_d - V_o)(T_s - t_{on}) = 0$$

$$V_d T_s = V_o (T_s - t_{on})$$

$$V_o = V_d \left[\frac{T_s}{T_s + t_{on}}\right] = V_d \left[\frac{1}{1-D}\right] \quad ①$$

where $D = \frac{t_{on}}{T_s}$

Assuming a lossless Circuit

Power in = Power OUT

$$V_d I_D = V_o I_O \Rightarrow \frac{I_o}{I_D} = \frac{V_d}{V_o}$$

OR $\frac{I_o}{I_D} = (1-D)$

Where I_D = average input current

what does I_D look like?

$$I_D = i_L(t)$$

so $I_D = \frac{I_1 + I_2}{2}$ ② – Avg input current

so $I_D = \frac{I_o}{1-D}$ --- from before so

$$\frac{I_1 + I_2}{2} = \frac{I_0}{1-D} \quad (3)$$

Now using $V_L = L\frac{di}{dt}$

$$i_L(t) = \frac{1}{L}\int V_L\, dt + \text{i.c.}$$

$$I_2 = \frac{1}{L}\int_0^{t_{on}} V_d\, dt + I_1$$

$$I_2 = \frac{V_d t_{on}}{L} + I_1$$

$$(I_2 - I_1) = \frac{V_d t_{on}}{L} \quad (4)$$

$$\boxed{\begin{aligned} I_2 - I_1 &= \frac{V_d t_{on}}{L} \\ \frac{I_1 + I_2}{2} &= \frac{I_0}{1-D} \\ I_0 &= \frac{I_1 + I_2}{2} \\ V_0 &= V_d\left[\frac{1}{1-D}\right] \end{aligned}}$$

Next: Find the boundry where the Inductor current becomes discontinuous

$\Rightarrow I_1 = 0$

Solve for I_1

$$I_1 + I_2 = \frac{2I_o}{1-D} \quad (3)$$

$$- \quad I_2 - I_1 = \frac{V_d t_{on}}{L}$$

$$2I_1 = \frac{2I_o}{1-D} - \frac{V_d t_{on}}{L}$$

Solve for $I_1 = 0$

$$\frac{2I_o}{1-D} - \frac{V_d t_{on}}{L} = 0$$

$$I_o = \left(\frac{V_d t_{on}}{2L}\right)(1-D)$$

From equation 1, $1-D = \frac{V_d}{V_o}$

So we Need

$$I_o \geq \left(\frac{V_d t_{on}}{2L}\right)\left(\frac{V_d}{V_o}\right)$$

Summary

Boost Regulator in continuous mode

$$V_o = V_d\left[\frac{1}{1-D}\right] \qquad D = \frac{t_{on}}{T_s}$$

$$t_{on} + t_{off} = T_s$$

$$I_o = I_D[1-D]$$

$$\left(\frac{I_1+I_2}{2}\right) = I_D = \frac{I_o}{1-D}$$

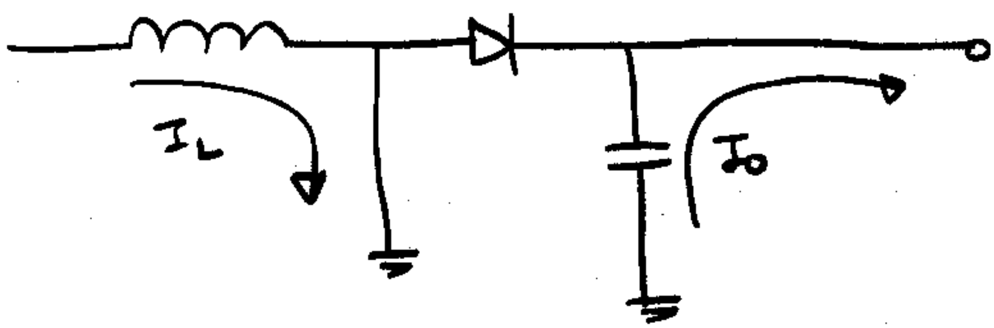
$$I_2 - I_1 = \frac{V_d\, t_{on}}{L}$$

$$I_o \geq \left(\frac{V_d\, t_{on}}{2L}\right)\left(\frac{V_d}{V_o}\right)$$; For continuous mode

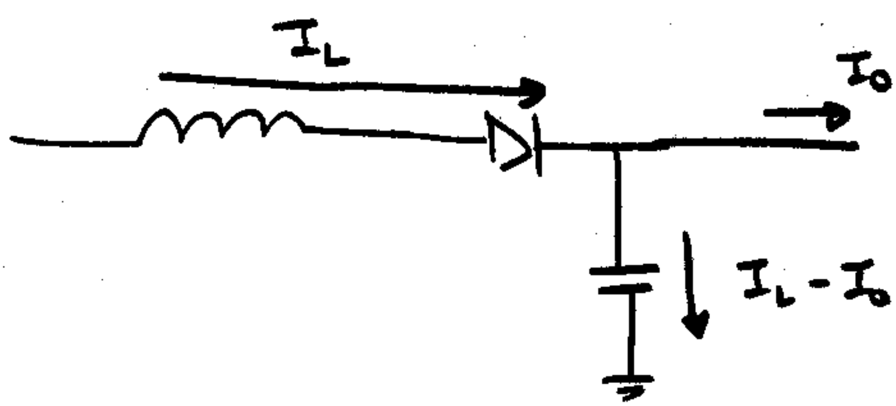
In continuous mode operation, the Boost regulator has a right half plane zero. This makes the boost Regulator diffucult to stabilize in continuous mode.

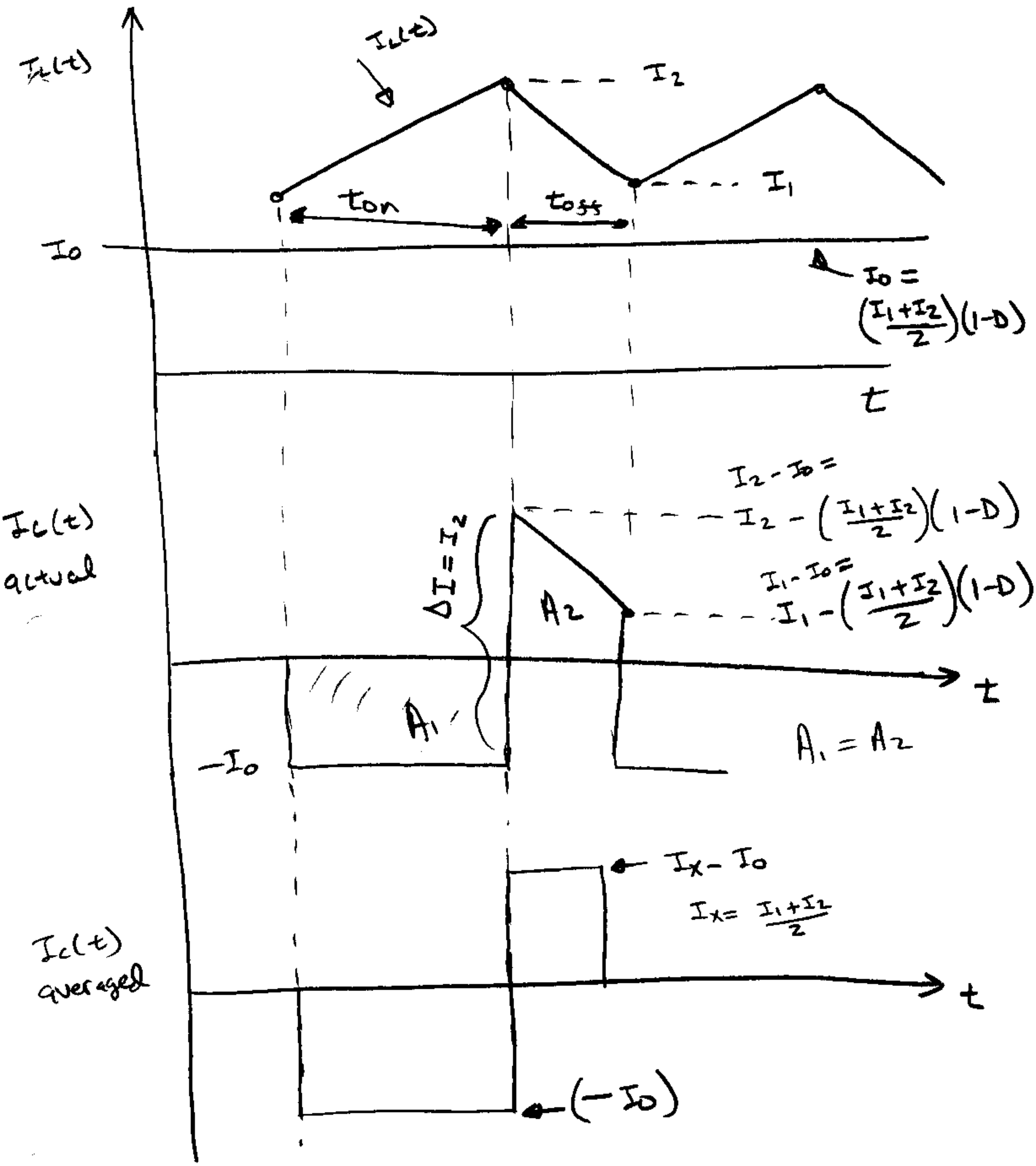
Capacitor Rms Ripple current for the continuous Boost converter

- During Ton, The switch is on and the capacitor must supply current I_o to the load



- During Toff, The switch is off. The inductor provides I_o to the output and also charges the capacitor



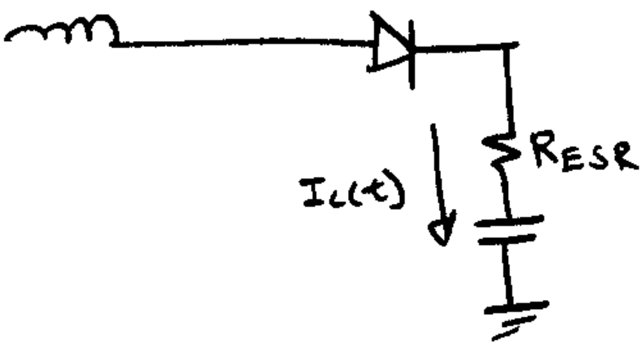


$I_L(t)$
$I_L(t)$
I_2
I_1
t_{on}
t_{off}
I_0
$I_0 = \left(\frac{I_1+I_2}{2}\right)(1-D)$
t
$I_c(t)$ actual
$I_2 - I_0 =$
$I_2 - \left(\frac{I_1+I_2}{2}\right)(1-D)$
$I_1 - I_0 =$
$I_1 - \left(\frac{I_1+I_2}{2}\right)(1-D)$
$\Delta I = I_2$
A_2
A_1
t
$-I_0$
$A_1 = A_2$
$I_x - I_0$
$I_x = \frac{I_1+I_2}{2}$
$I_c(t)$ averaged
t
$(-I_0)$

$$I_{Rms} = \sqrt{\frac{1}{T_S}\left[\int_0^{t_{on}} (-I_o)^2\,dt + \int_{t_{on}}^{T_s} (I_x - I_o)^2\,dt\right]}$$

Ripple Voltage due to $ESR = \Delta I\, R_{ESR}$

$$V_{RR} = I_Z R_{ESR}$$



Ripple due to change in capacitor voltage

$$q = CV$$

$$\Delta V = \frac{\Delta q}{C} = \frac{I_o\, t_{on}}{C}$$ ← put charge into C

EE 456

Boost Regulator Design - Continuous Mode Operation

$\mu s \equiv 10^{-6} \cdot sec$

Specify Input Voltage $V_D := 5 \cdot volt$

Specify Output Voltage $V_o := 50 \cdot volt$

Specify Switching Frequency $F_S := 20 \cdot kHz$

$T_S := \frac{1}{F_S}$ $\quad T_S = 50 \circ \mu s$

Specify the Assumed Efficiency $\quad Eff := 90\ \%$

Specify the Max output Current

The output Power is $\quad P_{out} := \frac{10 \cdot watt}{Eff}$

The output current is $\quad I_o := \frac{P_{out}}{V_o}$ $\quad I_o = 0.222 \circ amp$

Find Ton and Toff

$T_{off} := 1 \cdot \mu s$ $\quad T_{on} := 1 \cdot \mu s$

Given

$$\frac{T_{on}}{T_S} = \frac{V_o - V_D}{V_o}$$

$$T_{on} + T_{off} = T_S$$

$\begin{bmatrix} T_{on} \\ T_{off} \end{bmatrix} := Find(T_{on}, T_{off})$ $\quad T_{on} = 45 \circ \mu s$ $\quad T_{off} = 5 \circ \mu s$

$D := \frac{T_{on}}{T_S}$ $\quad D = 90 \circ \%$

Find the range of Inductors that will operate in continuous mode

Specify the minimum current we want the supply to operate in the continuous mode

$$I_{min} := \frac{I_o}{10}$$

$$L := \frac{V_D \cdot T_{on}}{2 \cdot I_{min}} \cdot \frac{V_D}{V_o}$$

For Continuous Mode, We need L greater than $L = 506.25 \circ \mu H$

Choose the Inductor $L := 1 \cdot mH$

Find the Min and max inductor currents

$I_1 := 1 \cdot amp$ $I_2 := 1 \cdot amp$

Given

$$\frac{I_1 + I_2}{2} = \frac{I_o}{1 - D}$$

$$I_1 - I_2 = V_D \cdot \frac{T_{on}}{L}$$

$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} := \mathrm{Find}(I_1, I_2)$ $I_1 = 2.335 \circ amp$ $I_2 = 2.11 \circ amp$

Choose the filter capacitor using Ripple voltage due to ESR and capacitor change in charge.

Specify the ripple voltage $V_{CR} := 20 \cdot mV$

$$ESR := \frac{V_{CR}}{I_1} \qquad ESR = 8.566 \cdot 10^{-3} \circ \Omega$$

For all electrolytic caps, assume that ESR*C=80µs

$$C_{ESR} := \frac{80 \cdot \mu s}{ESR} \qquad C_{ESR} = 9.339 \cdot 10^{3} \circ \mu F$$

Choose the capacitor using the ripple voltage due to the change in charge

$$C_q := \frac{I_o \cdot T_{on}}{V_{CR}} \qquad C_q = 500 \circ \mu F$$

Choose Choose a standard cap value greater than both calculations

$$C := 10000 \cdot \mu F$$

Calculate the Capacitor RMS Ripple Current

$$I_x := \frac{I_1 + I_2}{2} \qquad I_x = 2.222 \circ amp$$

$$I_{rms} := \sqrt{\frac{1}{T_S} \cdot \left[\int_{0 \cdot sec}^{T_{on}} I_o^{\,2}\, dt + \int_{T_{on}}^{T_S} \left(I_o - I_x\right)^2 dt \right]}$$

$$I_{rms} = 0.667 \circ amp$$

Summary

$L = 1\cdot 10^{3} \circ \mu H$ $\quad I_2 = 2.11 \circ amp$ $\quad I_1 = 2.335 \circ amp$ $\quad \dfrac{I_1 + I_2}{2} = 2.222 \circ amp$

$T_{on} = 45 \circ \mu s$ $\quad T_{off} = 5 \circ \mu s$

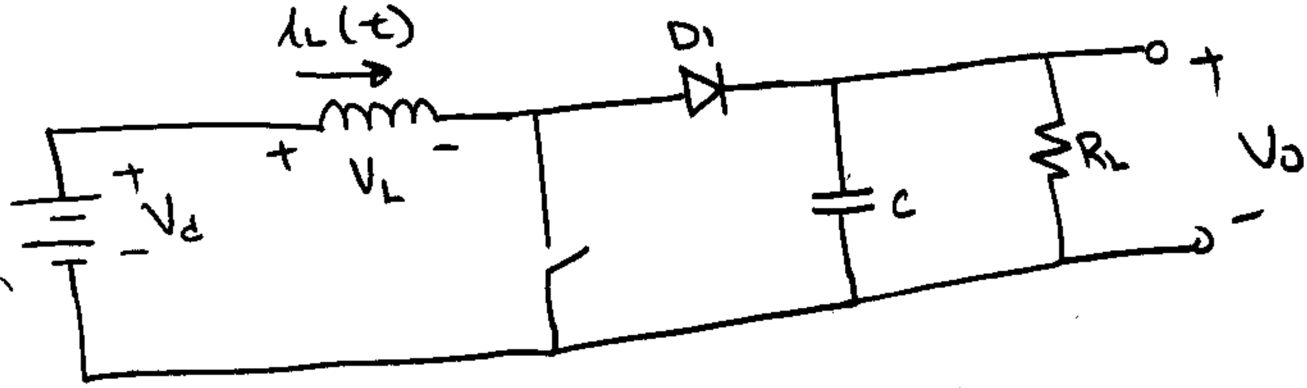
$V_D = 5 \circ volt$ $\quad V_o = 50 \circ volt$ $\quad I_o = 0.222 \circ amp$

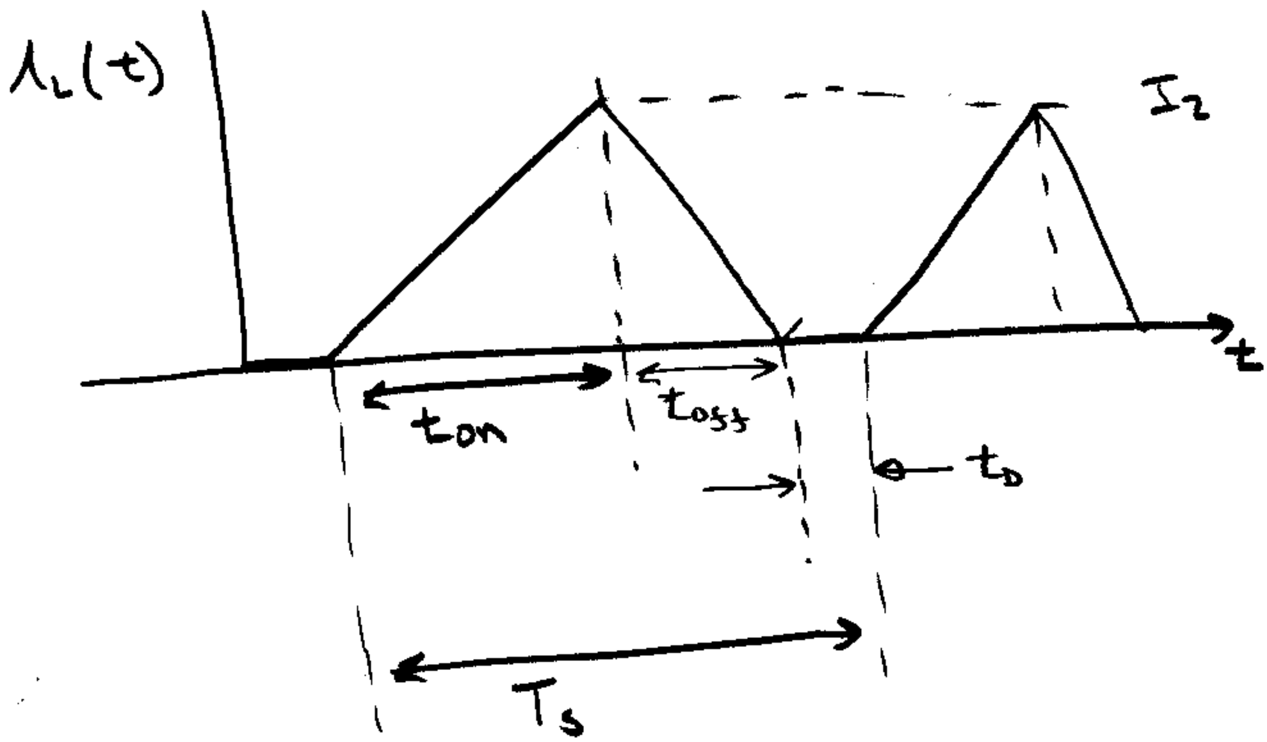
$C = 1\cdot 10^{4} \circ \mu F$ $\quad V_{CR} = 20 \circ mV$ $\quad I_{rms} = 0.667 \circ amp$

Cap Voltage = $V_o = 50 \circ volt$

Boost Regulator

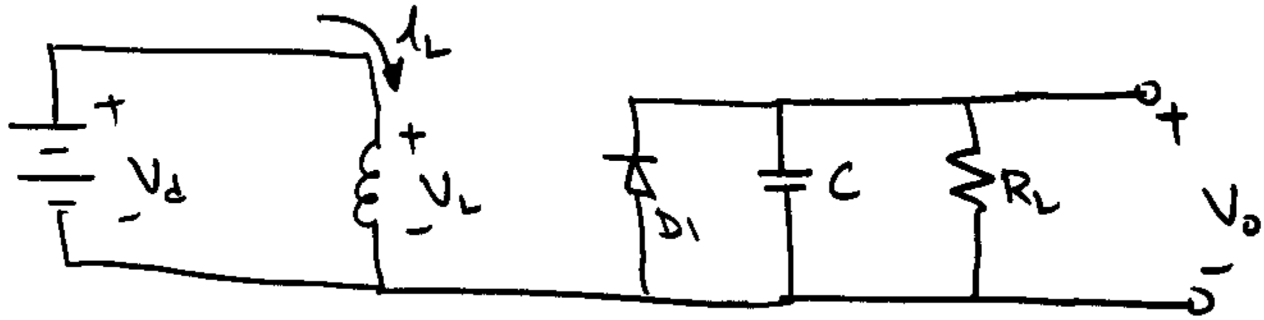
Discontinuous mode





t_D = dead Time

During t_{on} we have



C supplies power to R_L

D_1 = off

$V_L = V_d$

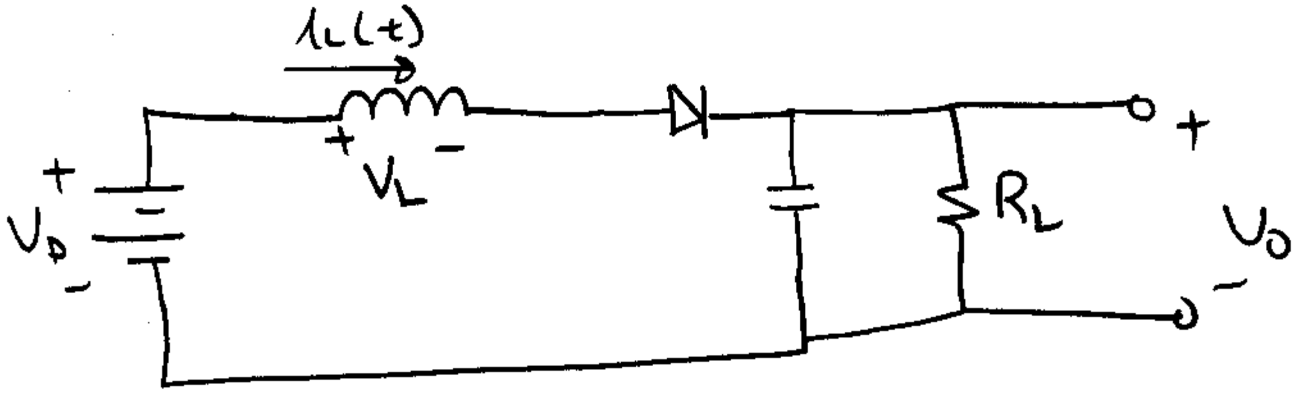
for the inductor: $V_L = L \dfrac{d i_L(t)}{dt}$

$$i_L(t) = \frac{1}{L} \int_0^{t_{on}} V_L(t)\, dt$$

OR

$$I_2 = \frac{V_d\, t_{on}}{L} \qquad (1)$$

During t_{off} we have



- Diode is ideal
- D_1 = on
- $V_L = V_d - V_o$; $V_L < 0 \Rightarrow i_L(t) \downarrow$

For the inductor: $V_L = L \dfrac{d\, i_L(t)}{dt}$

$$i_L(t) = \frac{1}{L} \int_{t_1}^{t} V_L(t)\,dt + I_2$$

OR

$$0 = \frac{1}{L} \int_{t_{on}}^{t_{on}+t_{off}} (V_d - V_o)\,dt + I_2$$

OR $I_2 = \dfrac{V_o - V_d}{L} t_{off}$ (2)

To ensure discontinuous mode

Choose $t_d = 0.2\,T_s$

$$\Rightarrow \boxed{t_{on} + t_{off} \leq 0.8\,T_s} \quad (3)$$

Power balance: Avg power delivered by source

$P_D = V_d \langle i_L \rangle$

During $t_{on} + t_{off}$ $\langle \hat{i}_L \rangle = \frac{I_2}{2}$

so over T_s

$$\langle i_L \rangle = \langle \hat{i}_L \rangle \left(\frac{t_{on} + t_{off}}{T_s} \right)$$

$$= \frac{I_2}{2} \left[\frac{t_{on} + t_{off}}{T_s} \right]$$

OR

$$P_D = \frac{V_d I_2}{2} \left[\frac{t_{on} + t_{off}}{T_s} \right]$$

and from equation ①, $I_2 = \frac{V_d t_{on}}{L}$

$$P_D = \frac{V_d^2 t_{on}}{2 L T_s}\left[t_{on} + t_{off}\right]$$

output power: $P_o = V_o I_o$

so $P_o = P_D$

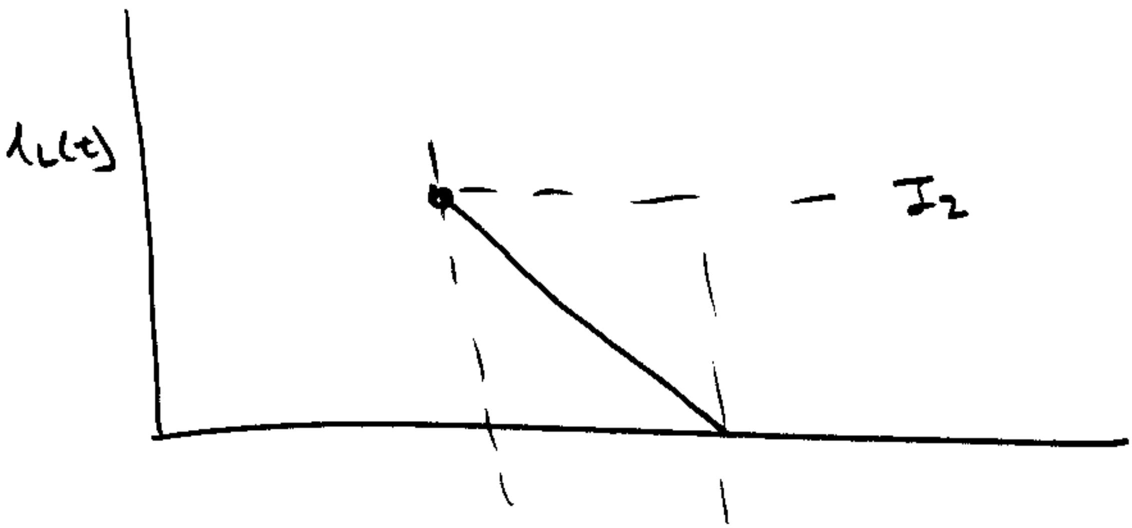
$$\boxed{V_o = \frac{V_d^2 t_{on}}{2 L T_s I_o}\left[t_{on} + t_{off}\right]} \quad (4)$$

Energy balance

- Energy dissipated in Load for Entire cycle

$$\mathcal{E}_o = V_o I_o T_s$$

During t_{off}



- all energy stored in L is delivered to Load
- Source V_d delivers energy to Load

$$\mathcal{E}_L = \frac{1}{2} L I_2^2$$

$$I_2 = \frac{V_d t_{on}}{L}$$

$$\Rightarrow \mathcal{E}_L = \frac{1}{2L} V_d^2 t_{on}^2$$

Now Find Energy delivered by the source

$$\mathcal{E}_s = V_d \langle I_d \rangle t_{off}$$

$$\langle I_d \rangle = \frac{I_2}{2} = \frac{V_d t_{on}}{2L}$$

so

$$\mathcal{E}_s = V_d \langle I_d \rangle t_{off} = \frac{V_d^2 t_{on}}{2L} t_{off}$$

so for energy balance

$$\mathcal{E}_L + \mathcal{E}_s = \mathcal{E}_o$$

$$\frac{1}{2L} V_d^2 t_{on}^2 + \frac{1}{2L} V_d^2 t_{on} t_{off} = V_o I_o T_s$$

OR

$$\frac{1}{2L} V_d^2 t_{on} \left[t_{on} + t_{off} \right] = V_o I_o T_s$$

Same as power balance.

Boost Regulator Summary

Discontinuous mode

$$I_a = \frac{V_d\, t_{on}}{L}$$

$$I_a = \left(\frac{V_o - V_d}{L}\right) t_{off}$$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad \frac{t_{on}}{t_{off}} = \frac{V_o - V_d}{V_d}$$

$$t_{on} + t_{off} \leq 0.8\, T_s$$

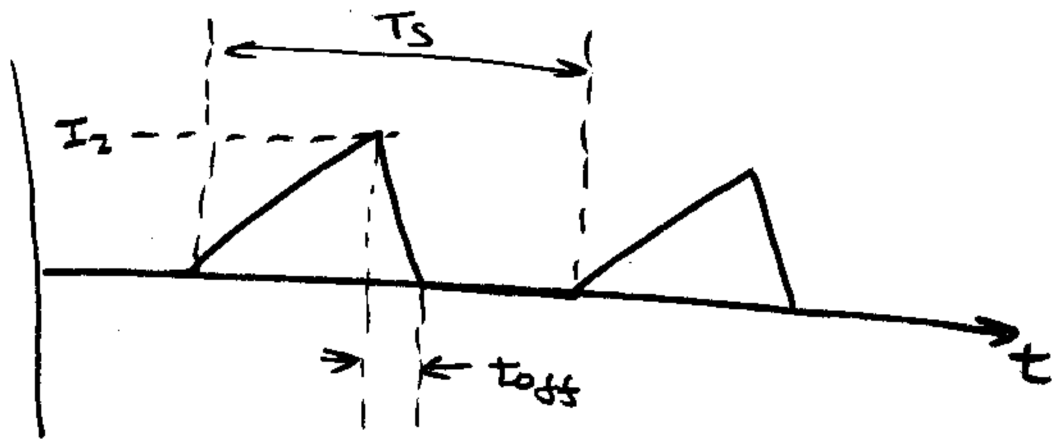
$$V_o = \frac{V_d^2\, t_{on}}{2 L T_s I_o}\left[t_{on} + t_{off}\right]$$

$$I_o \leq \left(\frac{V_d\, t_{on}}{2L}\right)\left(\frac{V_d}{V_o}\right)$$; for discontinuous mode

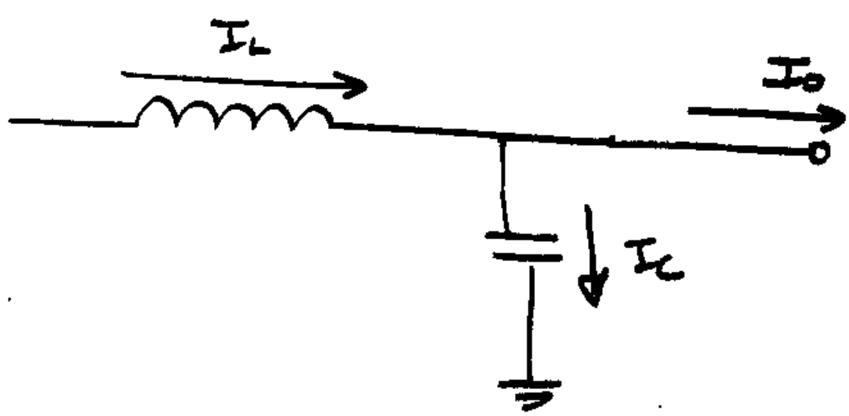
Boost Converter Discontinuous mode

Capacitor Rms Ripple current

— Inductor current



- During t_{off}, The inductor supplies I_o and charges the Capacitor

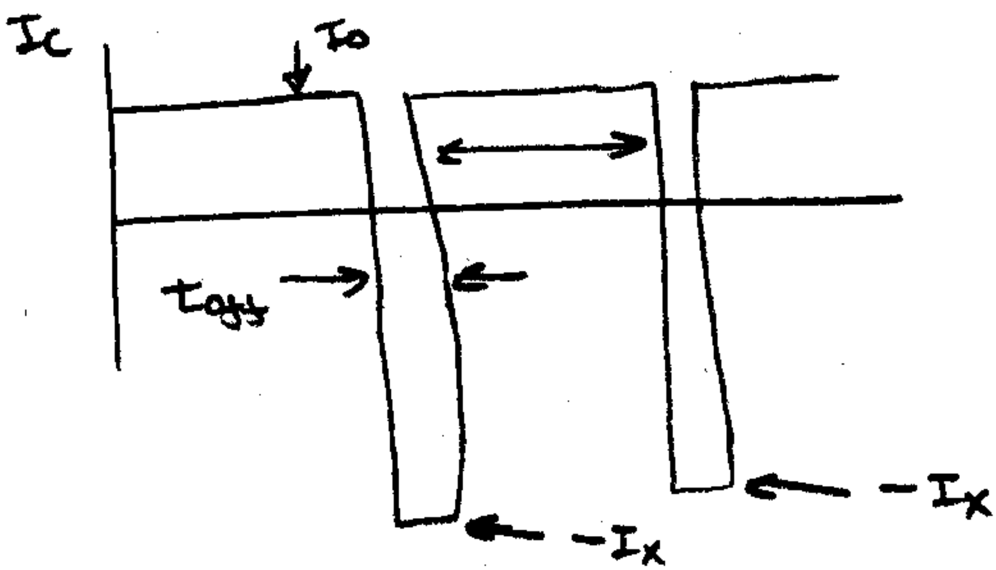


$$I_C = I_L - I_o$$

During t_{off}, the average inductor current is

$$I_x = \frac{I_2}{2}$$

Capacitor current



$$I_{RMS} = \sqrt{\frac{1}{T_S}\left[\int_0^{T_S - t_{off}} I_o^2\,dt + \int_0^{t_{off}} (-I_x)^2\,dt\right]}$$

EE 456

Boost Regulator Design - Discontinuous Mode Operation

Specify Output Voltage $V_o := 50 \cdot \text{volt}$ $\mu S \equiv 10^{-6} \cdot \text{sec}$

Specify Input Voltage $V_D := 5 \cdot \text{volt}$

Specify Switching Frequency $F_S := 20 \cdot \text{kHz}$

$$T_S := \frac{1}{F_S} \qquad T_S = 5 \cdot 10^{-5} \circ \text{sec}$$

Specify the Max Output Power and Efficiency $P_{out} := 10 \cdot \text{watt}$ $Eff := 90 \cdot \%$

Calculated the power for the design

$$P_{design} := \frac{P_{out}}{Eff} \qquad P_{design} = 11.111 \circ \text{watt}$$

Calculate the output current

$$I_{out} := \frac{P_{design}}{V_o} \qquad I_{out} = 0.222 \circ \text{amp}$$

Find Ton and Toff

$T_{off} := 1 \cdot \text{sec}$ $T_{on} := 1 \cdot \text{sec}$

Given

$$\frac{T_{on}}{T_{off}} = \frac{V_o - V_D}{V_D}$$

$$T_{on} + T_{off} = 0.8 \cdot T_S$$

$$\begin{bmatrix} T_{on} \\ T_{off} \end{bmatrix} := \mathrm{find}\left(T_{on}, T_{off}\right)$$

$T_{on} = 36 \cdot \mu S$ $\qquad$ $T_{off} = 4 \cdot \mu S$

Find the range of Inductors that will operate in discontinuous mode

$$L := \frac{V_D \cdot T_{on}}{2 \cdot I_{out}} \cdot \frac{V_D}{V_o}$$

For Discontinuous Mode, We need L less than $\qquad L = 40.5 \cdot \mu H$

FInd the inductor Value

$$L := \frac{{V_D}^2 \cdot T_{on}}{2 \cdot V_o \cdot T_S \cdot I_{out}} \cdot \left(T_{on} + T_{off}\right) \qquad L = 32.4 \cdot \mu H$$

Choose a standard size inductor $\qquad L := 30 \cdot \mu H$

FInd the peak current

$$I_2 := V_D \cdot \frac{T_{on}}{L} \qquad I_2 = 6 \circ \text{amp}$$

Choose the filter capacitor.

Assume that the major component of the ripple comes from the capacitor ESR

Specify the ripple due to the ESR $\quad V_{RR} := .05 \cdot \text{volt}$

$$ESR := \frac{V_{RR}}{(I_2)} \qquad ESR = 0.00833 \circ \Omega$$

For all electrolytic caps, assume that ESR*C=80μs

$$C := \frac{80 \cdot 10^{-6} \cdot \text{sec}}{ESR} \qquad C = 9.6 \cdot 10^{3} \circ \mu F$$

Choose the next size std capacitor $\quad C := 10000 \cdot \mu F$

$$ESR := \frac{80 \cdot 10^{-6} \cdot \text{sec}}{C}$$

Calculate the new ESR with the chosen capacitor

$$ESR = 8 \cdot 10^{-3} \circ \Omega$$

Calculate the Capacitor RMS Ripple Current

$$I_x := \frac{I_2}{2} \qquad I_x = 3 \cdot \text{amp}$$

$$I_{rms} := \sqrt{\frac{1}{T_S} \cdot \left[\int_{0 \cdot sec}^{T_{on}} I_{out}^2 \, dt + \int_{T_{on}}^{T_S} \left(I_{out} - I_x \right)^2 dt \right]}$$

$$I_{rms} = 1.482 \cdot \text{amp}$$

Summary

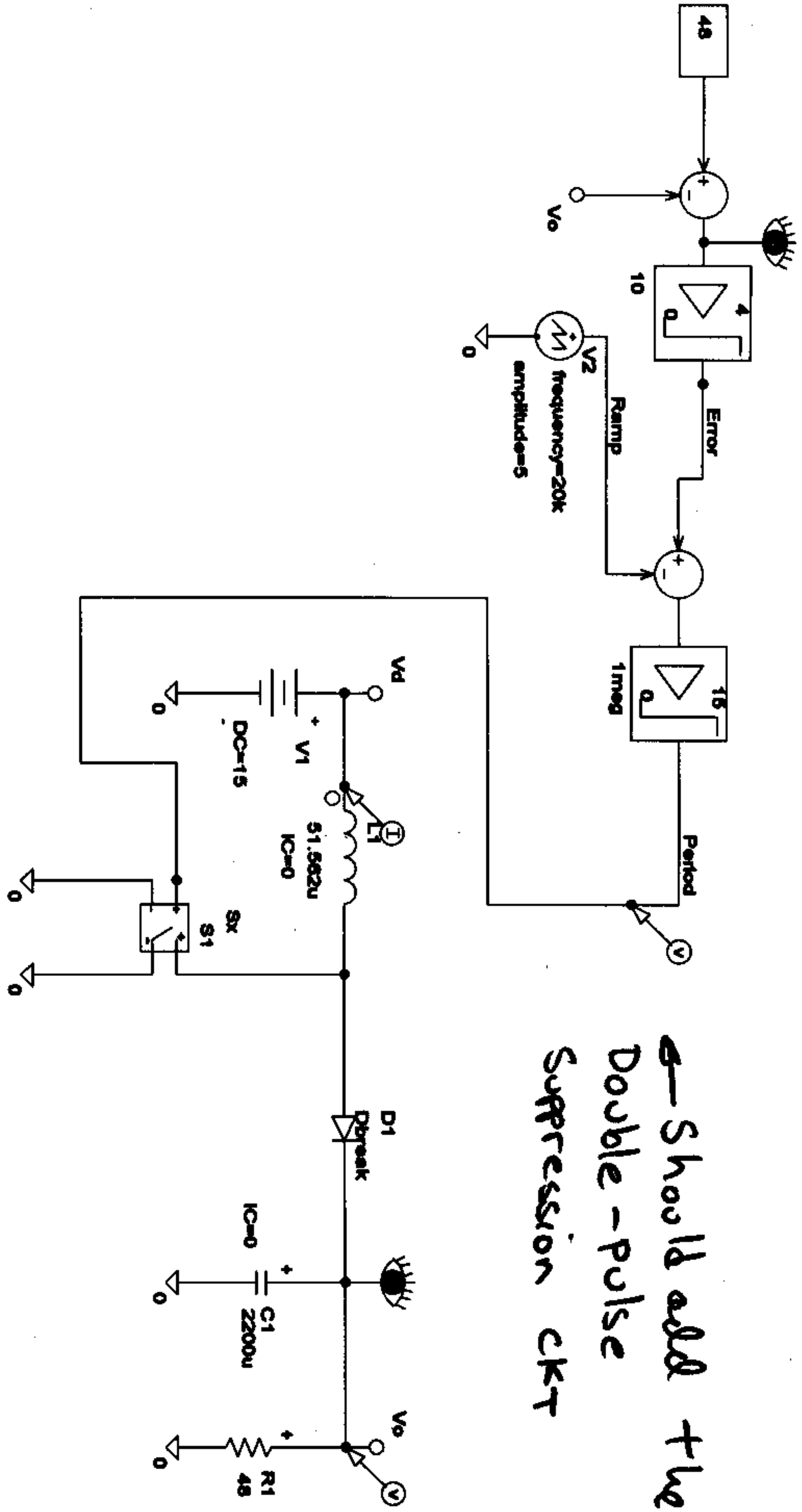
$L = 30 \cdot \mu H$ $\quad I_2 = 6 \cdot \text{amp}$

$T_{on} = 36 \cdot \mu S$ $\quad T_{off} = 4 \cdot \mu S$

$V_D = 5 \cdot \text{volt}$ $\quad V_o = 50 \cdot \text{volt}$ $\quad I_{out} = 0.222 \cdot \text{amp}$

$C = 1 \cdot 10^4 \cdot \mu F$ $\quad V_{RR} = 50 \cdot mV$ $\quad I_{rms} = 1.482 \cdot \text{amp}$

Cap Voltage $V_o = 50 \cdot \text{volt}$



← Should add the Double-Pulse Suppression CKT

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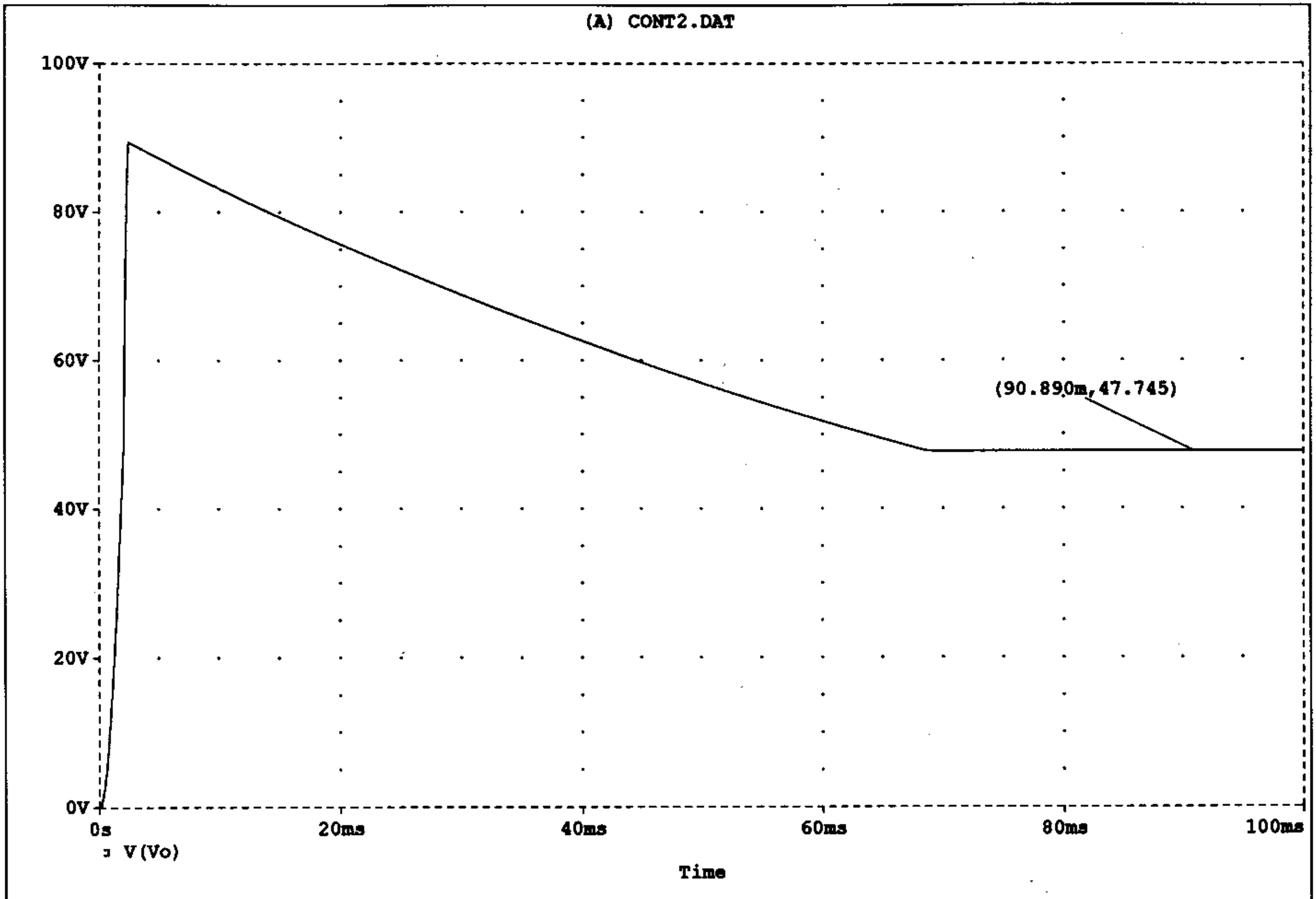

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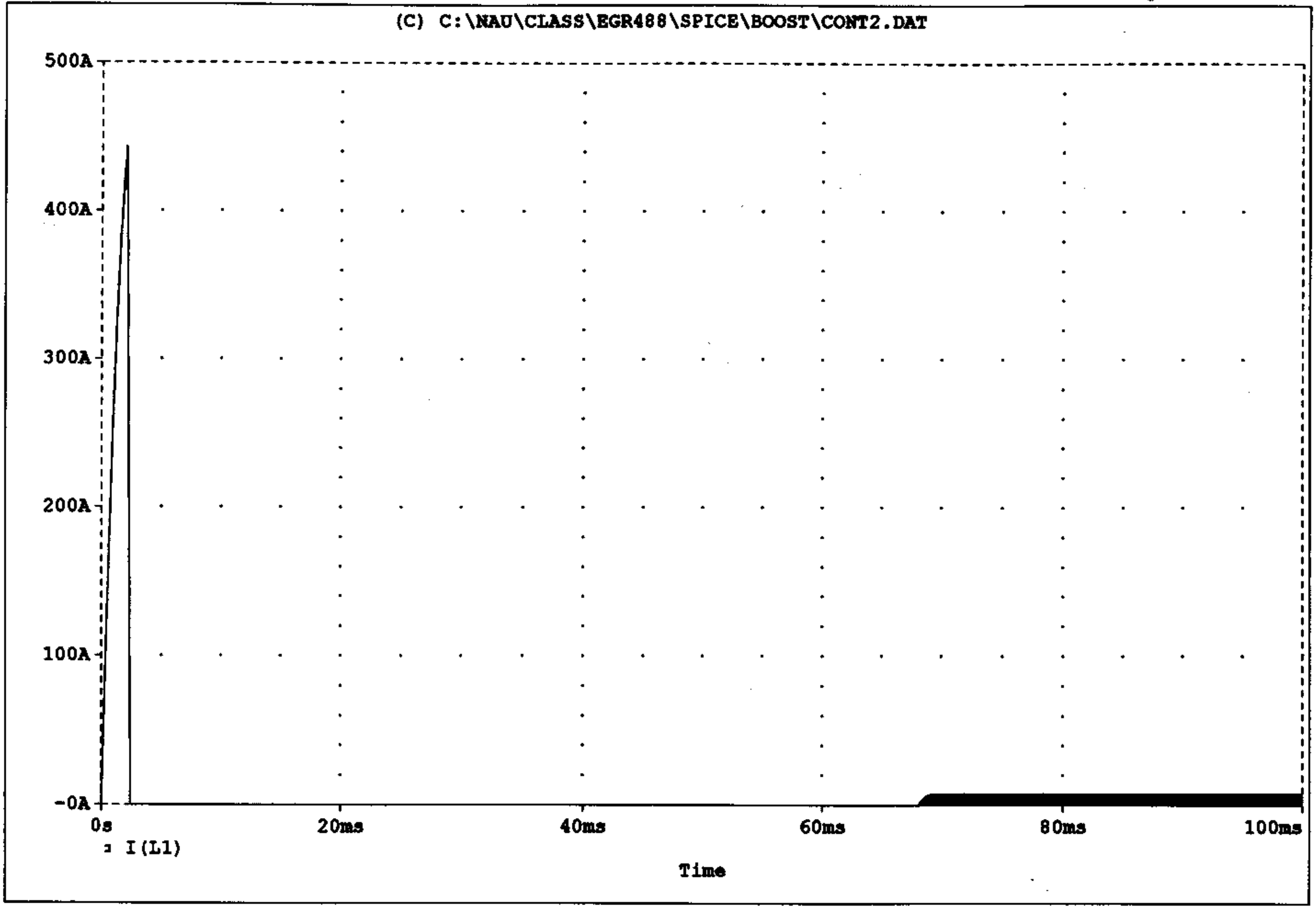

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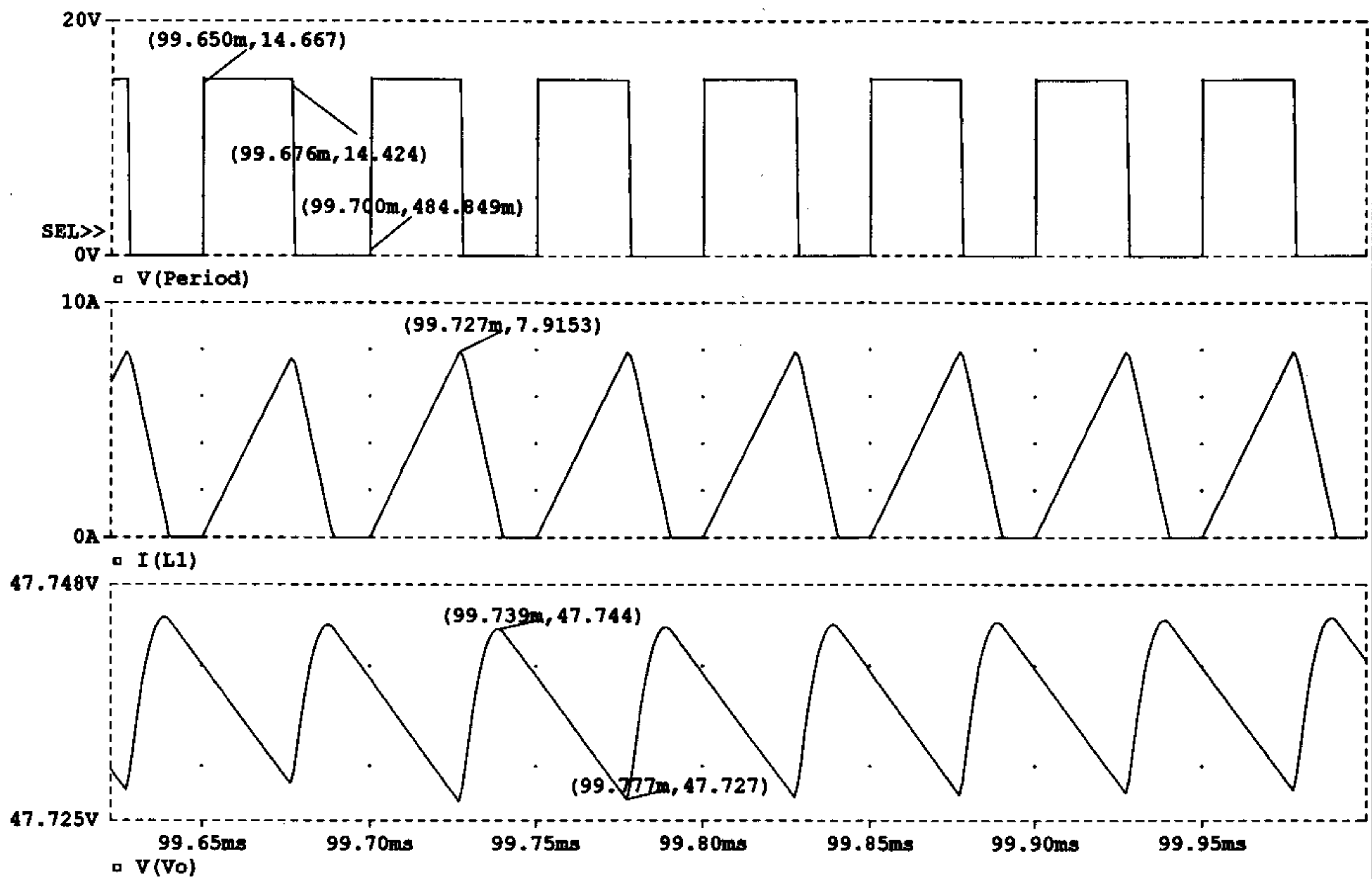

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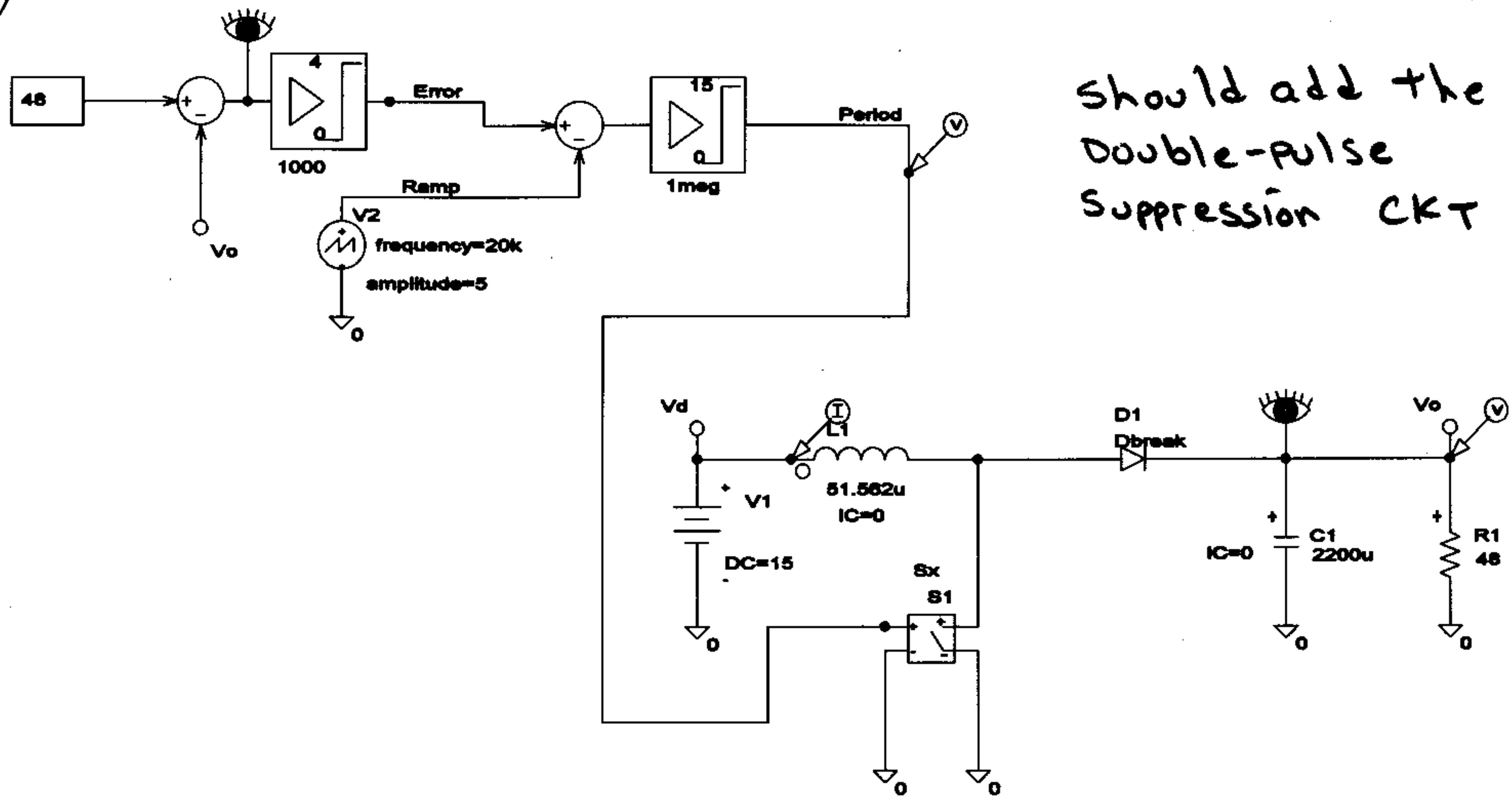

Should add the Double-Pulse Suppression CKT

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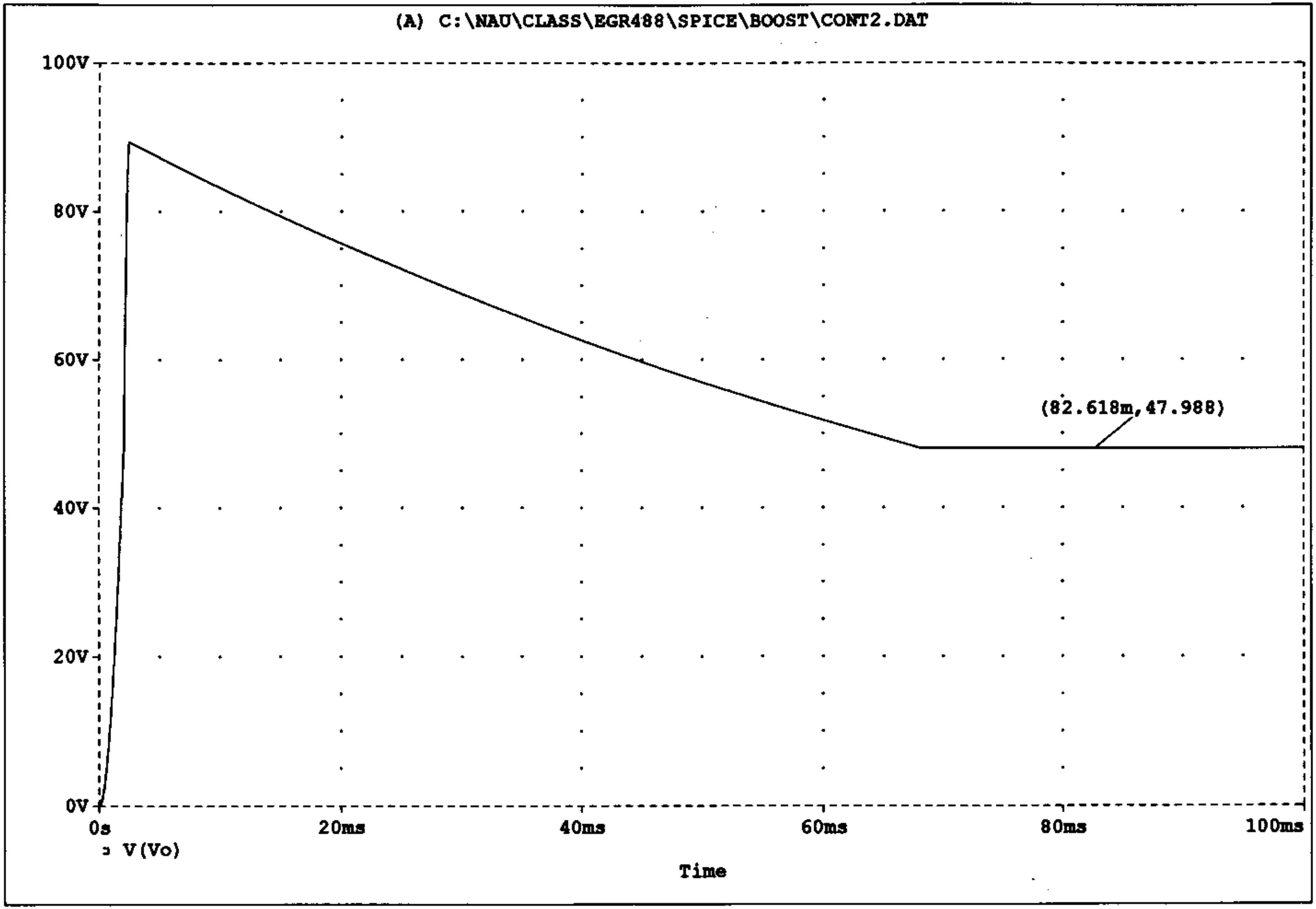

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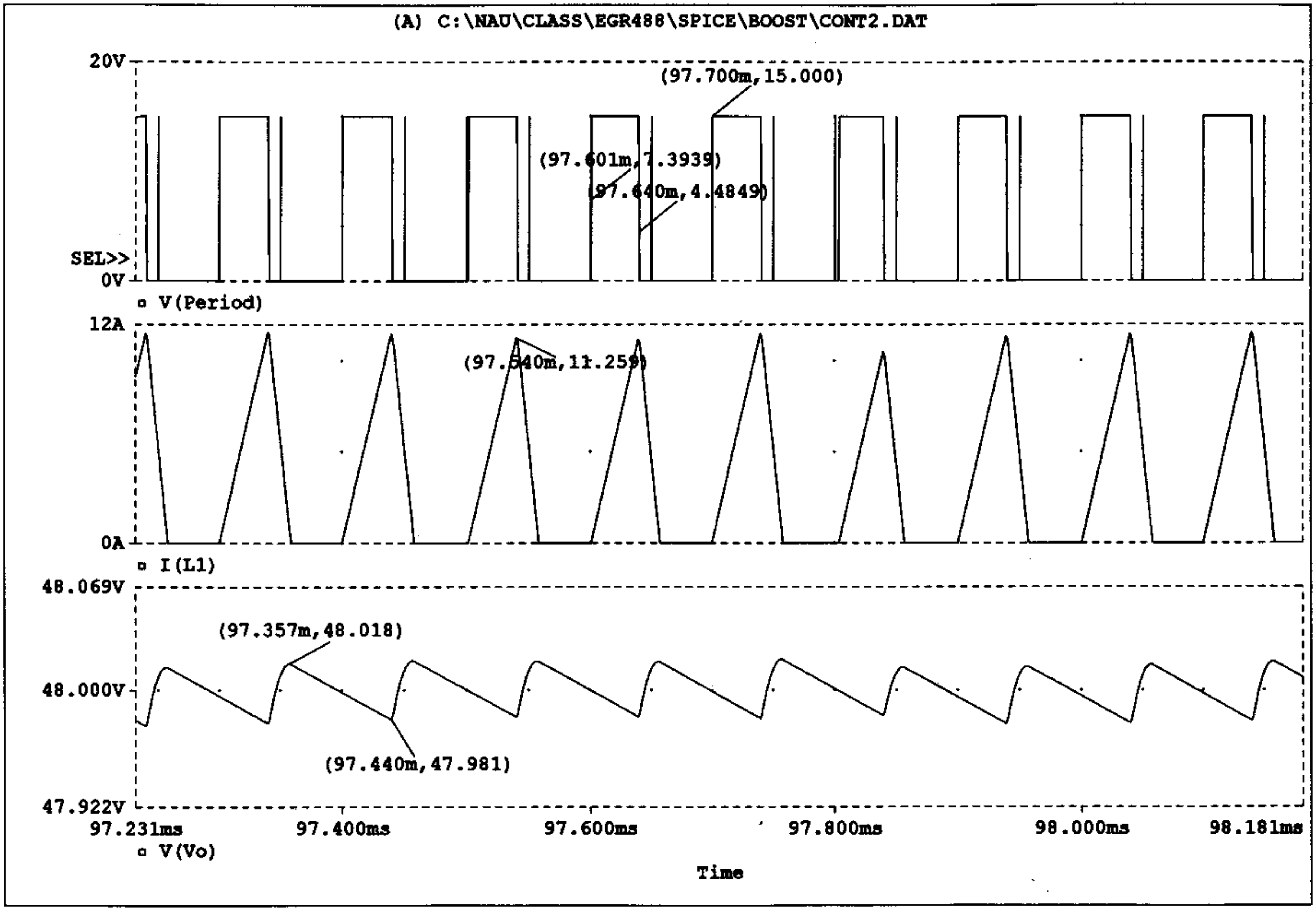
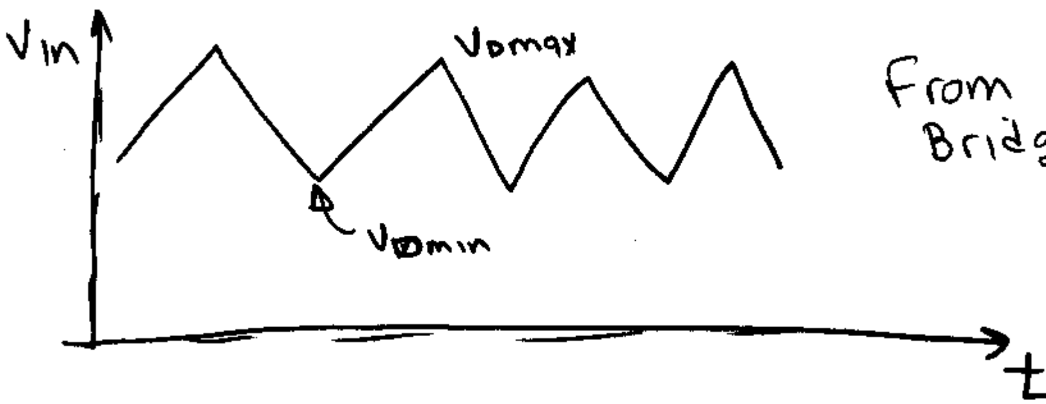
Date/Time run: 09/11/95 06:42:24 Temperature: 27.0



Date: September 11, 1995 Page 1 Time: 07:31:38

Worst Case Design

Variable Vin



From a Bridge Rect.

$$t_{on} = \frac{I_a L}{V_d}$$

⇒ The smaller V_d is, the longer it takes to reach I_D.

- we are usually concerned with energy transfer $\frac{1}{2} L I_a^2$
- we would like to charge L up to I_a. The smaller V_d is, the longer t_{on} must be to achieve I_a

⇒ Design with $V_{in\,min}$

$$t_{on\_max} = \frac{I_2 L}{V_{d\,min}}$$

Variable V_o (Variable Supply)

$$t_{off} = \frac{I_2 L}{(V_o - V_d)} \quad ; \quad V_o > V_d$$

- During t_{off} we must completely discharge the inductor.
- The smaller $|V_o - V_d|$, the longer t_{off} must be to discharge the inductor

$$t_{off\_max} = \frac{I_2 L}{(V_{omin} - V_{dmax})}$$

and

$$(t_{on\_max} + t_{off\_max}) = 0.8\, T_s$$